

# MSC MATHEMATICS ASSIGNMENT PAPERS

Part -I (PREVIOUS)

Paper -I, ALGEBRA

Max. Marks:20

1. Answer the following all questions.

8X2½=20

1. Let  $G$  be a group containing an element of finite order  $n > 1$  and exactly two conjugacy classes, then prove that  $|G| = 2$ .
2. Prove that Every finite group has a composition series
3. Prove that the centre of a ring is a subring
4. Let  $\{A_i\}_{i \in I}$  be a family of right ideals in a ring  $R$ . Then prove that  $\bigcap_{i \in I} A_i$  is also a right ideal.
5. If  $f: M \rightarrow N$  is an  $R$ -homomorphism of an  $R$ -module  $M$  to an  $R$ -module  $N$ , then prove that
  - i)  $\text{Ker}(f)$  is an  $R$ -submodule of  $M$
  - ii)  $I_m f$  is an  $R$ -submodule of  $N$
6. Let  $M$  be a finitely generated free module over a commutative ring  $R$ . Then prove that all bases of  $M$  are finite
7. Prove that every Boolean ring commutative and is of characteristic 2.
8. Define a Boolean algebra and give an example of it and also prove that in any Boolean algebra  $a^n = a$

# MSC MATHEMATICS ASSIGNMENT PAPERS

Part -I (PREVIOUS)

Paper -II, REALANALYSIS

Max. Marks:20

1. Answer the following all questions.

8X2½=20

1. Prove that closed subsets of compact sets are compact
2. Let  $f$  be a continuous mapping of a compact metric space  $X$  into a metric space  $Y$ . Then prove that  $f$  is uniformly continuous on  $X$
3. Define Riemann-Stieltjes integral and prove that

$$\int_a^b f d\alpha \leq \int_a^b f d\alpha \text{ with the usual notation}$$

4. Give an example to show that a convergent series of continuous functions may have a discontinuous sum
5. Define outer measure  $\mu^*$  and prove that  $\mu^*(E_1) \leq \mu^*(E_2)$  if  $E_1 \subset E_2$
6. Let  $X$  be a measurable space. Suppose  $f \in L^2(\mu)$  and  $g \in L^2(\mu)$ . Then prove that  $fg \in L(\mu)$  and  $\int_X |fg| d\mu \leq \|f\| \|g\|$

7. Show that  $\int_0^1 \frac{\log x}{\sqrt{x}} dx$  is Convergent, but  $\int_1^2 \frac{\sqrt{x}}{\log x} dx$  is divergent

8. Find the Fourier series of the periodic function  $f$  with period  $2\pi$ , defined by

$$f(x) = \begin{cases} 0 & \text{for } -\pi < x \leq 0 \\ x & \text{for } 0 \leq x \leq \pi \end{cases}$$

What is the sum of the series at  $x=0$