BRANCH : MATHEMATICS

Paper - I : ALGEBRA

(For D.D.E Students Only)

(Revised Regulations from 2009-2010)

(Regular/Supplementary)

Max. Marks : 20

SECTION - A

Answer any **Four** questions. Each question carries 5 marks.

1. Let G be a group containing an element of finite order $n>1$ and exactly two conjugacy classes. Prove that $|G|=2$.

2. Prove that a finite group G is a p-group if and only if its order is a power of p.

3. Let F be a field. Then prove that the characteristic of F is either O or a prime number p.

4. Prove that an irreducible element in a commutative principal ideal domain is always prime.

5. State and prove Schur's lemma.

6. Find the rank of the linear mapping $\phi:\mathbb{R}^4\to\mathbb{R}^3$, where $\phi(a,b,c,d)=(a+2b-c+d,-3a+b+2c-d,-3a+8b+c+d)$.

7. Prove that a bijective map of a lattice L into a lattice L^1 is a lattice isomorphism if and only if it and its inverse are order preserving.

8. Let $S=S_1\times S_2$ where $S_i, i=1,2$ are partially ordered and let μ,μ_1 *and* μ_2 be the Mobius functions of S, S_1, and S_2 respectively. Then prove that $\mu((x_1,x_2),(y_1,y_2))=\mu_1(x_1,y_1)\mu_2(x_2,y_2)$ for all $x_1,y_1\in S_1$ and $x_2,y_2\in S_2$.

SECTION - B

Answer One question from each unit. Each question carries 15 marks.

Unit-I

9. a) Prove that any two composition series of a finite group are equivalent.

   b) Let G be a nilpotent group. Then prove that every subgroup of G and every homomorphic image of G are nilpotent.

(OR)

10. a) Prove that the derived group of S_n is A_n.

    b) State and prove fundamental theorem of finitely generated abelian groups.

Unit-II

11. a) Let R be a ring such that $x^3 = x$ for all x in R. Prove that R is commutative.

    b) If R is a nonzero ring with unity 1, and I is an ideal in R such that I//R, then prove that there exists a maximal ideal M of R such that $I \subseteq M$.

(OR)

12. a) Prove that every euclidean domain is a PID.

    b) If. $f(x), g(x) \in R[x]$ then prove that $c(fg) = c(f)c(g)$. In particular, show that the product of two primitive polynomials is primitive.

Unit-III

13. a) Let M be an R-module. Then prove that Hom_R (M, M) is a subring of Hom (M, M).

    b) Let V be a nonzero finitely generated vector space over a field F. Then prove that V admits a finite basis.

(OR)

14. a) Prove that every finitely generated module is a homomorphic image of a finitely generated free module.

    b) Let A be an m×n matrix over F, and let Q be an m×n invertible matrix over F. Then prove that dim R (A) = dim R (AQ).

Unit-IV

15. a) Prove that a lattice L is modular if and only if whenever $a \geq b$ and $a \wedge c = b \wedge c$ and $a \vee c = b \vee c$ for some c in L, then a = b.

    b) State and prove Jordan-Holder-Dedekind theorem.

(OR)

16. a) Prove that the following two types of abstract systems are equivalent: Boolean algebra and Boolean ring.

    b) Describe the map coloring problem, and deduce the chromatic polynomial of Δ.

PART I - PREVIOUS

BRANCH : MATHEMATICS

Paper - II : REAL ANALYSIS

(FOR DDE Students Only)

(Revised Regulations from 2009-2010)

(Regular/Supplementary)

Max. Marks : 20

SECTION - A

Answer any **Four** questions. Each question carries 5 marks.

Prove that the set of all integers in countable.

Let $I = [0,1]$ be the closed unit interval suppose f is a continuous mapping of I into I. Prove that $f(x) = x$ for at least one x in I.

If $f(x) = 0$ for all irrational x, $f(x) = 1$ for all rational x then prove that f is not Riemann integrable on $[a,b]$ for any $a < b$.

4. State and prove fundamental theorem of calculus.

5. State and prove Factor's theorem.

6. If $f \in L(\mu)$ on E, then prove that $|f| \in L(\mu)$ on E, and $\left|\int_E f d\mu\right| \leq \int_E |f| d\mu$.

7. Show that the integral $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$ is convergent and hence evaluate it.

8. Show that $f(x,y) = y^2 + x^2 y + x^4$ has a minimum at (0, 0).

SECTION - B

Answer any One question from each unit. Each question carries 15 marks.

Unit-I

9. a) Let A be the set of all sequences whose elements are the digits 0 and 1. Prove that this set A is uncountable.

   b) Prove that every R-cell is compact.

(OR)

10. a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X.

[P.T.O.

b) Let f be monotonic on (a, b). Then prove that the set of points of (a, b) at which f discontinuous is at most countable. (

Unit-II

11. a) State and prove necessary and sufficient condition for the existence of Riemann Stieltjes integral. (

b) Suppose $f \in \Re(\alpha)$ on [a, b], $m \le f \le M$, ϕ is continuous of [m, M], and $h(x) = \phi(f(x))$ on [a, b]. Then prove that $h \in \Re(\alpha)$ on [a, b]. (

(OR)

12. a) Let $f \in \Re$ on [a, b]. For $a \le x \le b$, put $F(x) = \int_a^x f(t)\,dt$. Then prove that F continuous on [a,b]; furthermore, show that if f is continuous at a point x_0 of [a,b], th F is differentiable at x_0 and $F'(x_0) = f(x_0)$ (

b) Prove that there exists a real continuous function on the real line which is nowh differentiable. (

Unit-III

13. a) Let f and g be measurable real-valued functions defined on x, let F be real and continu on $\mathbb{R}^2$, and put $h(x) = F(f(x), g(x)), x \in X$. Then prove that h is measurable. particular, prove that $f+g$ and fg are measurable.

b) Suppose $f = f_1 + f_2$, where $f_i \in L(\mu)$ on E, i = 1,2. Then prove that $f \in L(\mu)$ or and $\int_E f d\mu = \int_E f_1 d\mu + \int_E f_2 d\mu$.

(OR)

14. a) State and prove Lebesque's dominated convergence theorem.

b) If $f \in \Re$ on [a,b], then prove that $f \in L$ on [a,b], and $\int_a^b f dx = \Re \int_a^b f dx$.

Unit-IV

15. a) Show that $\int_0^\infty \frac{\sin x}{x} dx$ is convergent.

b) Find the Fourier series of the periodic function f with period 2π, defined as foll

$$f(x) = \begin{cases} 0, & for \quad -\pi < 0 \le 0 \\ x, & for \quad 0 \le x \le \pi \end{cases}$$

What is the sum of the series at x = 0, $\neq \pi, 4\pi, -5\pi$?

(OR)

16\. a) Show that the function f, where $f(x,y)=\begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}, & if \quad x^2+y^2 \neq 0 \\ 0 \quad , & if \quad x=y=0 \end{cases}$ is continuous, possesses partial derivatives but is not differentiable at the origin.

b) Expand $x^2y+3y-2$ in powers of $(x-1)$ and $(y+2)$.

PART I - PREVIOUS

BRANCH : MATHEMATICS

Paper - III : DIFFERENTIAL EQUATIONS

(For D.D.E Students)

(Revised Regulations from 2010-2011)

(Regular/Supplementary)

Max. Marks : 20

SECTION - A

Answer any **Four** questions. Each question carries 5 marks.

1. Prove that there are three linearly independent solutions of the third order equation. $x''' + b_1(t)x'' + b_2(t)x' + b_3(t)x = 0$, $x \in I$ where I is an interval of the real line $\mathbb{R}$, where b_1, b_2 and b_3 are functions defined and continuous on I.

2. The motion of a simple pendulum is $x''(t) + k\sin x(t)$, where k is a constant. Find power series solution of this equation that satisfies the initial conditions $x(0) = \frac{\pi}{6}$ and $x'(0) = 0$.

3. Let A (t) be an n×n matrix which is continuous on I. Suppose a matrix Φ satisfies $X' = A(t)X$. Then prove that det Φ satisfies the first order equation $(\det\Phi)' = (tA)(\det\Phi)$.

4. Let f(t,x) be a continuous function defined over a rectangle $R = \{(t,x) : |t - t_0| \le p, |x - x_0| \le q\}$. Here p, q are some positive real numbers. Let $\frac{\partial f}{\partial x}$ be defined and continuous on R. Then prove that f (t, x) satisfies the Lipchitz condition in R.

5. Let $v, w \in C^1\{[t_0, t_0 + h], R\}$ be lower and upper solutions of $x' = f(t,x), x(t_0) = x_0$ respective where $f \in C[D, \mathbb{R}]$, where D is an open connected set in $\mathbb{R}^2$ and $(t_0, x_0) \in D$. Suppose that, for $x \ge y$, f satisfies the inequality $f(t,x) - f(t,y) \le L(x - y)$, where L is a positive constant. Then prove that $v(t_0) \le w(t_0)$ implies that $v(t) \le w(t)$, $t \in [t_0, t_0 + h]$

6. Describe Sturm-Liourille problem.

7. Find the general solution of the differential equation. $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$

8. Find the solution of the equation $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=x-y$.

SECTION - B

Answer any One question from each unit. Each question carries 15 marks.

Unit-I

9. a) State and prove Abel's formula.

b) Solve $x^{11}-x^1-2x=3e^t$, $x(0)=1, x^1(0)=0$ by using the method of Laplace transforms.

(8)

(OR)

10. a) Find power series solution of Legendre equation $(1-t^2)x^{11}-2tx^1+p(p-1)x=0$, where p is a real number.

b) Show that

i) $\frac{d}{dt}\left[t^pJ_p(t)\right]=t^pJ_{p-1}(t)$, and

ii) $\frac{d}{dt}\left[t^{-p}J_p(t)\right]=-t^{-p}J_{p+1}(t)$

Unit-II

11. a) Let $\Phi(t), t\in I$ denote a fundamental matrix of the system x^1=Ax such that $\Phi(0)=E$, where A is a constant matrix. Here E denotes the identify matrix. Then show that Φ satisfies $\Phi(t+s)=\Phi(t)\Phi(s)$ for all s, t in I.

b) State and prove F loquat theorem.

(OR)

12. a) State and prove the existence of solutions of the initial value problem $x^1f(t,x), x(t_0)=x_0$ on the ship S defined by $S=\{(t,x):|t-t_0|\le T \ and |x|<\infty\}$ in the large.

b) Let $x(t)=x(t,t_0,x_0)$ and $x^*(t)=x(t,t_0^*,x_0^*)$ be solutions of the IVPS $x'=f(t,x), x(t_0)=x_0$ and $x'=f(t,x), x(t_0^*)=x_0^*$ respectively on an interval $a\le t\le b$. Let $(t,x(t)),(t,x^*(t))$ lie in a domain D for $a\le t\le b$. Further, let $f\in Lip(D,k)$ be founded by L in D. Then, prove that for any $\epsilon>0$, there exists $\delta=\delta(\epsilon)>0$ such that $|x(t)-x^*(t)|<\epsilon$, $a<t<b$ whenever $|t_0-t_0^*|<\delta$ and $|x_0-x_0^*|<\delta$.

Unit-III

13. a) State and prove Bihari's inequality.

b) State and prove Alekseev's formula.

(OR)

14. a) Let y and z be linearly independent solutions of $L(x)=(px')'-qx=0$ where p, p^1 & q are real valued continuous functions on $a\le t\le b$. Define Green's function by

$$G(s,t)=\begin{cases}\dfrac{-y(t)z(s)}{A} & \text{if } t\le s\\[2mm] \dfrac{-y(s)z(t)}{A} & \text{if } t\ge s.\end{cases}$$

where $A=p(t)\left[y(t)z^1(t)-y^1(t)z(t)\right]$, $t\in[a,b]$, a nonzero constant. Then prove that x (t) is a solution of $L(x)+f(t)=0,\ a\le t\le b$, with boundary conditions

$$m_1x(a)+m_2x^1(a)=0$$

$$m_3x(b)+m_4x^1(b)=0$$

With the assumption that at least one of m_1 and m_2 and one of m_3 and m_4 are non zero, if and only if $x(t)=\int_a^b G(s,t)f(s)\,ds$

b) Show that the boundary value problem $x^{11}+\cos x=0, x(0)=x(1)=0$ and show that this BVP has a unique solution.

Unit-IV

5. a) Find the integral surface of the linear partial differential equation $x(y^2+z)p-y(x^2+z)q=(x^2-z^2)z$ which contains the straight line x+y=0, z=1.

b) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis.

(OR)

16. a) Find a particular integral of the equation $(D^2 - D')z = 2y - x^2$

b) Reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$ to canonical form and hence solve it.

M.Sc. DEGREE EXAMINATION — MARCH/APRIL 2021

Part I – Previous

Branch – Mathematics

Paper IV – COMPUTING TECHNIQUES

(Revised Regulations from 2009–2010)

Max. Marks 20

PART A

Answer any FOUR questions.

Each question carries 5 marks.

1. How do you debug a shell script?

2. Discuss the cubic spline method for a BVP with an example.

3. What is meant by control of Statements? Explain in brief about any two types of control statements.

4. Define arrays. Explain about different types of arrays.

5. What are (a) Linear Fredholm integral equations of the 1st and 2nd (LFIE) kinds (b) Voltera integral equations of the 2nd kind (c) homogeneous and nonhomogeneous (LEIE) questions (d) eigen values and eigen functions (e) singular and nonsingular kernels.

6. What is meant by pointers? How they are used?

7. Write short notes on I/O multiplexing.

8. Discuss the essential aspects of Unix System which make it unique in design.

PART B

Answer ONE question from each Unit.

Each question carry equal marks.

UNIT I

9. (a) Discuss in detail how the file processing is carried out in unix.

(b) Discuss in detail the directory structure in the unix file system.

Or

10. (a) Discuss in detail the looping system in Unix.

(b) Discuss in detail all those special Unix features with a software Engineering view point.

UNIT II

11. (a) Write a C program to compute (n)!

(b) Explain about Iterative statements with simple examples? Write a program to display "hello world".

Or

12. (a) Write a C program to print the Floyd's triangle.

1				
2	3			
4	5	6		
7	8	9	10	
11	12	13	14	15

(b) Write a note on structure, union and enum. Explain with an example.

UNIT III

13. (a) Discuss in detail the method of least squares.

(b) Fit a straight line $y = a + bx$ to the following data by the method of least squares. $(0,1), (1,3), (3,2), (6,5), (8,4)$.

Or

14. Use Runge-Kutta 4th order, find y for $x = 0.1, 0.2, 0.3$ given that $y' = xy + y^2$, $y(0) = 1$.

UNIT IV

15. Solve one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$; $y(0,t) = 0$; $y(l,t) = 0$, $y(x,0) = f(x)$ and $\frac{\partial y}{\partial t}(x,0) = 0 < x < l$.

Or

16. (a) Explain the cubic spline method in solving Fredholm integral equations.

(b) Solve $f(x) = \int_0^1 (x + t)\, f(t) dt = \frac{3}{2}x - \frac{5}{6}$.

PART I : PREVIOUS

BRANCH-MATHEMATICS

Paper - V : COMPLEX ANALYSIS

Revised Regulations from 2009-2010

(FOR DDE Students Only)

(Regular/Supplementary)

Max. Marks : 20

SECTION - A

Answer any **Four** questions. Each question carries 5 Marks.

1. Describe the construction of the stereographic projection.

2. State and Prove Liouville Theorem.

3. If f(z) is analytic and not identically zero in some domain D containing $z = z_0$ then prove that its zeroes are isolated.

4. Discuss the pole singularities of the function $f(z) = \dfrac{z^2 - 1}{z - 1}$

5. Evaluate $I = \dfrac{1}{2\pi i} \oint_{C_2} \left(\dfrac{3z+1}{z(z-1)^3} \right) dz$ where C_2 is the circle $|z| = 2$

6. Evaluate $h(x)$ where the Laplace transform of $h(x)$ is given by $\hat{H}(S) = \dfrac{1}{S(S^2+1)}$

7. State and Prove Schwarz symmetry principle.

8. Find all bilinear transformations that map 0 and 1 to 0 and 1 respectively.

SECTION - B

Answer **One** question from each Unit. All questions carry equal Marks.

UNIT-I

9. a) Determine where f(z) is analytic when $f(z) = (x + \alpha y)^2 + 2i(x - \alpha y)$ for α real and constant.

[P.T.O.

b) If f(z) is continuous in a simply connected domain D and if $\oint_C f(z)dz = 0$ for every simple closed contour C lying in D, then prove that there exists a function f(z), analytic in D, such that $F'(z) = f(z)$.

(OR)

10. a) State and Prove Cauchy's theorem.

b) State and Prove Cauchy's integral formula.

UNIT-II

11. a) State and Prove Taylor series theorem.

b) Find the Laurent expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ for $1 < |z| < 1$.

(OR)

12. a) State and Prove Mittag-Leffler-simple poles theorem.

b) Show that the product $\prod_{R=1}^{\infty} \left(1 - \frac{z^4}{R^4}\right)$ represents an entire function with zeros at $z = \pm R, \pm iR; R = 1, 2, \ldots\ldots$

UNIT-III

13. a) Evaluate $I = \int_{-\infty}^{\infty} \frac{x^2}{x^4+1} dx$

b) State and Prove Jordan lemma.

(OR)

14. a) Evaluate $I = \int_0^{\infty} \frac{dx}{x^3+a^3}$, $a > 0$

b) State and prove the Argument principle.

UNIT-IV

15. a) Assume that f(z) is analytic and not constant in a domain D of the complex z-plane. Then prove that for any point $z \in D$ for which $f'(z) \neq 0$, This mapping is conformal.

b) Assume that f(z) is analytic and not constant in a domain D of the complex z-plane. Suppose that $f'(z_0) = f''(z_0) = \ldots\ldots = f^{(n-1)}(z_0) = 0$, while $f^{(n)}(z_0) \neq 0$, $z_0 \in D$. Then prove that the mapping $Z \to f(z)$ magnifies n times the angle between two intersecting differentiable arcs that meet at (z_0).

(OR)

6. a) Show that the function $W = \int_0^z \left(\dfrac{dt}{(1-t^6)^{\frac{1}{3}}} \right)$ maps a regular hexagon into the unit circle.

b) Prove that a necessary and sufficient condition for a bilinear transformation to ma the disk $|z| < 1$ onto $|w| < 1$ is that it be of the form $w = \beta \dfrac{z-\alpha}{\bar{\alpha} z - 1}$, $|\beta| = 1$, $|\alpha| <$
